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A Method to Calculate the Aircraft Ground Minimum Control Speed Based on Mathematical Simulation

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Abstract

The airworthiness compliance of the ground minimum control speed (V_{MCG}) must be shown before the type certificate of a transport category airplane is issued under CCAR-25. A method to calculate V_{MCG} based on mathematical simulation is provided. All forces and moments act on the aircraft in the takeoff run are considered in this method. The model of the pilot's operation in airworthiness compliance flight test of V_{MCG} is established in this method, and the time interval between the instant at which the critical engine is failed and the instant at which the pilot recognizes and reacts to the engine failure is considered in the model. Based on this method, V_{MCG} is obtained through the mathematical simulation of the takeoff run of an aircraft after the critical engine is failed. V_{MCG} calculated by the method presented in this paper can be used to guide the airworthiness design and V_{MCG} flight tests of an aircraft.

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Keywords: Aircraft, Airworthiness compliance showing, Ground Minimum control speed, Mathematical simulation, Ground run

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1. Introduction

Minimum control speed on the ground (V_{MCG}) of an aircraft is prerequisite to determine the speeds such as takeoff speed V_1 under CCAR-25 [1]. The transport category airplanes airworthiness standards such as CCAR-25, FAR-25 and JAR-25, require that the airplanes must show their airworthiness compliance of V_{MCG} before the Type Certificate of Airworthiness is issued [2, 3].

The airworthiness compliance of V_{MCG} should eventually be shown by flight test [1]. However, in the design of an aircraft, it is necessary to calculate V_{MCG} as accurately as possible based on the relevant data. By analyzing the calculated V_{MCG}, appropriate modification or improvement can be made in the design of aircraft, so as to ensure that V_{MCG} of the aircraft can meet the requirements of the airworthiness standards. In addition, the flight test of the airworthiness compliance show of V_{MCG} is a high-risk subject during the test flight. Prior to the V_{MCG} flight test, it is important to design the test program based on the accurately calculated V_{MCG} so as to reduce the risk and time of the flight test.

At present, V_{MCG} is generally calculated based on the balance equations of aircraft lateral and yaw forces (moments) [4-6]. However, there are following problems in this method.

① The method does not considered the process of the thrust decrease after the critical engine is failed. And the time interval between the instant at which the critical engine is failed and the instant at which the pilot recognizes and reacts to the engine failure is not taken into account. However, these factors are all important to V_{MCG}.

② The track and other motion parameters of the aircraft can not be calculated by the balance equations of lateral and yaw forces (moments). So this method can not express whether the maximum lateral deviate distance of the aircraft from the centerline of the runway will exceed 9 meters or not.

③ Since the operating engines are at maximum available takeoff power or thrust during the takeoff run, The aircraft is in a state of speeding up. However, the balance equations can not reflect the speed changes of the aircraft after the critical engine is failed.

Therefore, error will exists between the V_{MCG} calculated by the balance equations and the practical V_{MCG} obviously.

In response to the aforementioned problems, this paper will introduce an aircraft dynamic model of takeoff run. A model of the pilot's operation for airworthiness compliance flight test of V_{MCG} will be presented. Then the "aircraft-pilot" closed-loop motion model will be established. And the takeoff run of an aircraft can be accurately mathematic simulated by this “aircraft-pilot” model. Based on the “aircraft-pilot” model, this paper will present a method to calculate V_{MCG} precisely through the accurate mathematical simulation of the aircraft takeoff run.

2. Analysis of airworthiness standard

China Civil Aviation Regulations Part 25 (CCAR-25) 25.149 (e) provides the requirements of determining V_{MCG}.[1]. During the takeoff run, the yaw moment is strong after the critical engine is failed due to the large thrust of the operating engines. However, since the speed of the aircraft is low, the available control moment of the rudder is small. With the increasing speed of the aircraft in the takeoff run, the available rudder control moment also increases. When at a certain speed, the critical engine is suddenly made inoperative, then use the rudder control alone, as limited by 667N of control force, and the lateral control to the extent of keeping the wings level to enable the takeoff to be safely continued using normal piloting skill. The path of aircraft from the point at which the critical engine is made inoperative to the point at which the aircraft recover to a direction parallel to the centerline of the runway is completed deviates 9 meters laterally from the centerline at the deviatest point, the speed at which the critical engine fails is V_{MCG} of the aircraft under the validation test conditions.

CCAR 25.149 does not provide the limitation of V_{MCG}. However, in 25.107, V_{MCG} is used to determine the takeoff speed V_1.

3. Dynamic model of aircraft takeoff run

This paper assumes that the aircraft landing gear and tires are rigid. The lateral and torsional movement of the landing gear is ignored. And the aircraft pitch, roll and vertical movement of the aircraft center of gravity (CG) caused by the retractable landing gear and tires compression is also ignored [7, 8]. Transform all the forces and moments act on the aircraft to the aircraft CG, the dynamic equations of the ground run of a tricycle aircraft can be represented as follows

$$\begin{cases} m\dot{V}_k = A_{xk} + T_{xk} + G_{xk} + W_{xk} \\ mV_k\dot{\chi} = A_{yk} + T_{yk} + G_{yk} + W_{yk} \\ 0 = A_{zk} + T_{zk} + G_{zk} + W_{zk} \\ 0 = L_A + L_T + L_G + L_W \\ 0 = M_A + M_T + M_G + M_W \\ I_z\dot{r} = N_A + N_T + N_G + N_W \end{cases} \tag{1}$$

Where, m is the mass of the aircraft; V_k is the aircraft run speed; superscript "·" denotes the derivative of time t; A, T, G, W are respectively the aerodynamic force, engine thrust, gravity force and the ground force act on the aircraft; subscripts xk, yk and zk denote the component forces along x, y, z axis in the path coordinate system [9, 10]; L, M and N are respectively the rolling moment, pitching moment and yawing moment, all these moments are defined in the body coordinate system [9, 10], and the effect points of these moments are all located at the aircraft CG; subscript A, T, G and W denote the moments caused by aerodynamic force, engine thrust, gravity force and ground force act on the aircraft respectively; I_z is the inertia of yaw moment; r is the yaw rate.

3.1. Aerodynamic force

The projection vector of the aerodynamic force in the airflow coordinate system [9, 10] is

$$\vec{A}_a = \begin{bmatrix} -D \\ C \\ -L_{force} \end{bmatrix} \tag{2}$$

Where, D, C and L_{force} are respectively the aerodynamic drag force, lateral force and lift force act on the aircraft. The projection vector of the aerodynamic force in the path coordinate system is

$$\vec{A}_k = \begin{bmatrix} A_{xk} \\ A_{yk} \\ A_{zk} \end{bmatrix} = L_{ka} \cdot \vec{A}_a \tag{3}$$

Where, L_{ka} is the transformation matrix from the airflow to the path coordinate system [9, 10].

The projection vector of the aerodynamic moments in the body coordinate system can be calculated by

$$\vec{M}_{Ab} = \begin{bmatrix} L_A \\ M_A \\ N_A \end{bmatrix} = \begin{bmatrix} C_l \cdot \frac{1}{2}\rho \cdot V_a^2 \cdot S \cdot b \\ C_m \cdot \frac{1}{2}\rho \cdot V_a^2 \cdot S \cdot c \\ C_n \cdot \frac{1}{2}\rho \cdot V_a^2 \cdot S \cdot b \end{bmatrix} \tag{4}$$

Where, C_l, C_m and C_n are the aerodynamic rolling moment coefficient, pitching moment coefficient and yaw moment coefficient respectively, the reference points of all these moment coefficients are located at the aircraft CG; ρ is air density; V_a is airspeed; S is wing area; b is wingspan; c is the mean aerodynamic chord of the aircraft.

The projection vector of the airspeed in the body coordinate system is

$$\vec{V}_{ab} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = L_{bk} \begin{bmatrix} V_k \\ 0 \\ 0 \end{bmatrix} - L_{bg} \begin{bmatrix} u_{wg} \\ v_{wg} \\ w_{wg} \end{bmatrix} \tag{5}$$

Where, L_{bk} is the transformation matrix from the path to the body coordinate system [9, 10]; L_{bg} is the transformation matrix from the ground to the body coordinate system [9, 10]; $[u_{wg} \quad v_{wg} \quad w_{wg}]^T$ is the projection vector of the wind speed in the ground coordinate system.

Airspeed V_a, aerodynamic attack angle α and sideslip angle β can be calculated by equation *(6)*.

$$\begin{cases} V_a = \sqrt{\left(u^2 + v^2 + w^2\right)} \\ \alpha = \arctan\left(w/u\right) \\ \beta = \arcsin\left(v/V_a\right) \end{cases} \tag{6}$$

Based on the status parameters such as V_a, α and β, the aerodynamic data and the deflection of control surfaces, the aerodynamic force vector $\vec{A}_k$ and moment vector $\vec{M}_{Ab}$ can be calculated by equation *(2)* to *(4)*.

3.2. Engine thrust

The force vector $\vec{T}_b$ is the projection of the engine thrust in the body coordinate system, that is

$$\vec{T}_b = \begin{bmatrix} \sum_{i=1}^{n} T_i \cdot \cos\phi_T \\ 0 \\ -\sum_{i=1}^{n} T_i \cdot \sin\phi_T \end{bmatrix} \tag{7}$$

Where, n is the quantity of the aircraft engines; T_i is the thrust of the i th engine, if the inoperative engine lead to added drag force, a negative T_i is used to express the added drag force; ϕ_T is the engine installation angle.

The force vector $\vec{T}_k$ is the projection of the engine thrust in the coordinates of the path, that is

$$\vec{T}_k = \begin{bmatrix} T_{xk} \\ T_{yk} \\ T_{zk} \end{bmatrix} = L_{kb} \cdot \vec{T}_b \tag{8}$$

Where, L_{kb} is the transformation matrix from the body to path coordinate system [9, 10].

The projection vector of the moments caused by the engine thrust in the body coordinate system is

$$\vec{M}_{Tb} = \begin{bmatrix} L_T \\ M_T \\ N_T \end{bmatrix} = \begin{bmatrix} -\sum_{i=1}^{n}\left(T_i \cdot y_{Ti} \cdot \sin\phi_T\right) \\ -l_T \cdot \sum_{i=1}^{n} T_i \\ -\sum_{i=1}^{n}\left(T_i \cdot y_{Ti} \cdot \cos\phi_T\right) \end{bmatrix} \tag{9}$$

Where, y_{Ti} is the distance from the thrust line of the i th engine to the aircraft CG projected on y-axis of the body coordinate system, and y_{Ti} is positive when the i th engine is on the right of the aircraft CG; l_T is the pitch arm of the engine thrust, l_T is positive when the thrust line is above the aircraft CG.

The thrust T_i of each engine can be obtained based on the experiment data and the failure conditions of the engines. Then the force vector $\vec{T}_k$ and the moment vector $\vec{M}_{Tb}$ caused by the engine thrust can be calculated by equation *(7)*, *(8)* and *(9)*.

3.3. Gravity

The force vector $\vec{G}_g$ is the gravity force of the aircraft projected in the ground coordinate system

$$\vec{G}_g = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \tag{10}$$

Where, g is the acceleration of gravity.

The force vector $\vec{G}_k$ is the aircraft gravity force projected in the coordinates of the path, that is

$$\vec{G}_k = \begin{bmatrix} G_{xk} \\ G_{yk} \\ G_{zk} \end{bmatrix} = L_{kg} \cdot \vec{G}_g \tag{11}$$

Where, L_{kg} is the transformation matrix from the ground to the path coordinate system [9, 10].

Since the aircraft CG is the origin of the body coordinate system, the moments caused by the aircraft gravity projected in the body coordinate system is

$$\vec{G}_{Ab} = \begin{bmatrix} L_G \\ M_G \\ N_G \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

3.4. Ground forces

With equation *(1)*, W_{zk} , L_W and M_W can be written as equation *(13)*.

$$\begin{cases} W_{zk} = -T_{zk} - A_{zk} - G_{zk} \\ L_W = -L_T - L_A - L_G \\ M_W = -M_T - M_A - M_G \end{cases} \tag{13}$$

By analyzing the forces of the ground act on the aircraft wheels, W_{zk} , L_W and M_W can be calculated by

$$\begin{cases} W_{zk} = -N_{mL} - N_{mR} - N_n \\ L_W = N_{mL} \cdot y_{mL} - N_{mR} \cdot y_{mR} - N_n \cdot y_n - \left(F_{mLy} + F_{mRy} + F_{ny}\right) \cdot z_{cg} \\ M_W = -\left(N_{mL} + N_{mR}\right) \cdot x_m + N_n \cdot x_n + \left(F_{mLx} + F_{mRx} + F_{nx}\right) \cdot z_{cg} \end{cases} \tag{14}$$

Where, N_{mL} , N_{mR} and N_n are respectively the supportive forces of the ground act on the left main wheel, right main wheel and nose wheel; y_{mL} is the distance between the left main wheel and the aircraft CG projected on the y-axis of the body coordinate system; y_{mR} is the distance between the right main wheel and the aircraft CG projected on the y-axis of the body coordinate system; y_n is the distance between the nose wheel and the aircraft CG projected on the y-axis of body coordinate system, and y_n is positive when the nose wheel is on the right of the aircraft CG; F_{mLy} , F_{mRy} and F_{ny} are respectively the y-axis component forces of the ground act on the left main wheel, right main wheel and nose wheel in the body coordinate system; F_{mLx} , F_{mRx} and F_{nx} are respectively the x-axis component forces of the ground act on the left main wheel, right main wheel and nose wheel in the body coordinate system; z_{cg} is the vertical distance between the ground and aircraft CG; x_m is the projection distance between the main wheel and aircraft CG on the x-axis of the body coordinate system; x_n is the projection distance between the nose wheel and aircraft CG on the x-axis of the body coordinate system.

The horizontal forces of the ground act on the aircraft wheels can be shown as Fig. 1.

The frictions of the ground act on the wheels can be calculated by equation *(15)*, *(16)* and *(17)* [8].

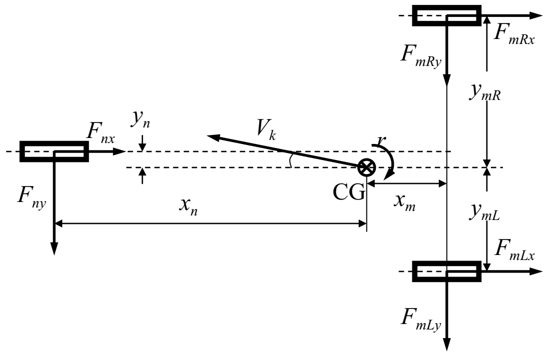

Fig. 1. Horizontal forces act on the wheels

$$\begin{cases} V_{mLx} = V_k \cdot \cos\beta_k + y_{mL} \cdot r \\ V_{mLy} = V_k \cdot \sin\beta_k - x_m \cdot r \\ \beta_{mL} = \arctan\left(\dfrac{V_{mLy}}{V_{mLx}}\right) \\ F_{mLx} = -\mu \cdot N_{mL} \\ F_{mLy} = -K_{mL} \cdot \beta_{mL} \cdot N_{mL} \end{cases} \tag{15}$$

$$\begin{cases} V_{mRx} = V_k \cdot \cos\beta_k - y_{mR} \cdot r \\ V_{mRy} = V_k \cdot \sin\beta_k - x_m \cdot r \\ \beta_{mR} = \arctan\left(\dfrac{V_{mRy}}{V_{mRx}}\right) \\ F_{mRx} = -\mu \cdot N_{mR} \\ F_{mRy} = -K_{mR} \cdot \beta_{mR} \cdot N_{mR} \end{cases} \tag{16}$$

$$\begin{cases} V_{nx} = V_k \cdot \cos\beta_k - y_n \cdot r \\ V_{ny} = V_k \cdot \sin\beta_k + x_n \cdot r \\ \beta_n = \arctan\left(\dfrac{V_{ny}}{V_{nx}}\right) - \delta_n \\ F_{nx} = -\mu \cdot N_n \cdot \cos\delta_n + K_n \cdot \beta_n \cdot N_n \cdot \sin\delta_n \\ F_{ny} = -\mu \cdot N_n \cdot \sin\delta_n - K_n \cdot \beta_n \cdot N_n \cdot \cos\delta_n \end{cases} \tag{17}$$

Where, μ is the rolling friction coefficient of the ground effecting on the wheels; β_k is the sideslip angle of the path; V_{mLx}, V_{mRx} and V_{nx} are the x-axis component velocity in the body coordinate system of the left main wheel, right main wheel and nose wheel respectively; V_{mLy}, V_{mRy} and V_{ny} are the y-axis component velocity in the body coordinate system of the left main wheel, right main wheel and nose wheel respectively; β_{mL} and β_{mR} are the sideslip angles at the point of left main wheel and right main wheel respectively; β_n is the sideslip angle of the nose wheel; δ_n is the swiveling angle of the nose wheel, δ_n is positive when the nose wheel is swiveled to the right; K_{mL}, K_{mR} and K_n are the sliding friction coefficients of the ground effecting on the left main wheel, right main wheel and nose wheel respectively.

The relationship between K_{mL} and β_{mL} can be shown in Fig. 2, and the relationships between K_{mR} and β_{mR}, K_n and β_n are in common with Fig. 2 [8].

Based on equation *(13)* to *(17)*, the forces of the ground act on the aircraft wheels can be calculated by equation *(18)*.

In equation *(18)*, the superscript "+" denotes the Moore-Penrose generalized inverse of a matrix [11].

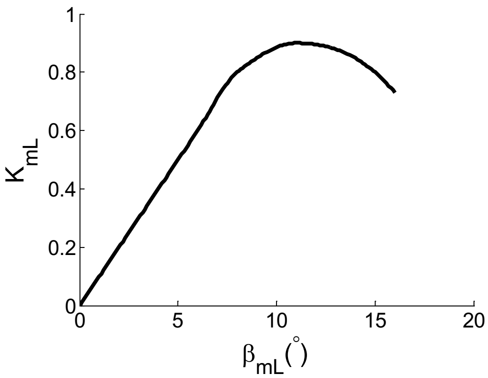

Fig. 2. Sliding friction coefficient of the ground acts on the wheel

$$\begin{bmatrix} N_{mL} \\ N_{mR} \\ N_n \\ F_{mLx} \\ F_{mRx} \\ F_{nx} \\ F_{mLy} \\ F_{mRy} \\ F_{ny} \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ y_{mL} & -y_{mR} & -y_n & 0 & 0 & 0 & -z_{cg} & -z_{cg} & -z_{cg} \\ -x_m & -x_m & x_n & z_{cg} & z_{cg} & z_{cg} & 0 & 0 & 0 \\ K\cdot\beta_{mL} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \mu & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & K\cdot\beta_{mR} & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \mu & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu\cdot\sin\delta_n + K_n\cdot\beta_{n-s}\cdot\cos\delta_n & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \mu\cdot\cos\delta_n - K_n\cdot\beta_{n-s}\cdot\sin\delta_n & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^{+} \begin{bmatrix} -T_{zk} - A_{zk} - G_{zk} \\ -L_T - L_A - L_G \\ -M_T - M_A - M_G \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (18)$$

The force vector $\vec{W}_k$ is the ground forces act on the aircraft projected in the path coordinate system. It can be calculated by

$$\vec{W}_k = \begin{bmatrix} W_{xk} \\ W_{yk} \\ W_{zk} \end{bmatrix} = L_{kb} \cdot \begin{bmatrix} F_{mLx} + F_{mRx} + F_{nx} \\ F_{mLy} + F_{mRy} + F_{ny} \\ -(N_{mL} + N_{mR} + N_n) \end{bmatrix} \qquad (19)$$

The projection vector of the moments caused by the ground forces act on the aircraft in the body coordinate system $\vec{M}_{Wb}$ can be calculated by

$$\vec{M}_{Wb} = \begin{bmatrix} L_W \\ M_W \\ N_W \end{bmatrix} = \begin{bmatrix} -L_T - L_A - L_G \\ -M_A - M_T - M_G \\ -F_{mLy}x_m - F_{mRy}x_m + F_{ny}x_n + F_{mLx}y_{mL} - F_{mRx}y_{mR} - F_{nx}y_n \end{bmatrix} \qquad (20)$$

With the forces and moments act on the aircraft (such as A_{xk}, T_{xk}, N_A, N_T, etc.) calculated from equation *(2)* to *(20)*, $\dot{V}_k$, $\dot{\chi}$ and $\dot{r}$ can be calculated by equation *(1)*. Then, we can obtain V_k, χ and r through the time integration of $\dot{V}_k$, $\dot{\chi}$ and $\dot{r}$. Since the pitching and rolling motion of the aircraft are ignored during the three-wheel run. We can get others motion parameters of the aircraft according to the relationships between the motion parameters. And then, the ground run of the aircraft can be mathematical simulated.

4. Pilot's operation model in V_{MCG} airworthiness compliance showing

During the takeoff run, since the air aerodynamic pressure is low, the rudder hinge moment is minor. And the actuator is used in the rudder loop. So, we can hold that the rudder pedal force is less than 667N when the rudder is full deflected. Therefore, in order to ensure that the path of the aircraft from the point at which the critical engine is made inoperative (at the centerline of the runway) to the point at which the aircraft recovers to a direction parallel to the centerline is completed does not deviate more than 9 meters (30 feet) laterally from the centerline at any point, the pilot should set the rudder full deflected to the side without engine failure at the moment of recognizing the critical engine failure. And, in order to keep the aircraft wing level, aileron should be controlled appropriate by the pilot while the rudder is full deflected. At the same time, the pilot should stick forward appropriately to maintain the nose wheel touch the ground slightly [12]. In addition, to simulate the suddenly failure of the critical engine, the critical engine should be set to idling or throttle fuel cut off, and the operating engines are all at maximum available takeoff power or thrust [1]. Therefore, in the V_{MCG} airworthiness compliance show, the pilot's operation after the critical engine failure can be expressed as follows

$$\delta_{rc} = \begin{cases} 0 & t_{es} < t \le t_{es} + t_{pd} \\ \delta_{r-full} & t_{es} + t_{pd} < t \le t_1 \end{cases} \tag{21}$$

$$\delta_{ac} = \begin{cases} 0 & t_{es} < t \le t_{es} + t_{pd} \\ \delta_{a1} & t_{es} + t_{pd} < t \le t_1 \end{cases} \tag{22}$$

$$\delta_{ec} = \begin{cases} \delta_{e1} & N_n > N_{n\min} \\ \delta_{e2} & N_n \le N_{n\min} \end{cases} \tag{23}$$

$$\begin{cases} \delta_{p-c} = \delta_{p-io} & t > t_{es} \\ \delta_{p-uc} = \delta_{p-matp} \end{cases} \tag{24}$$

Where, δ_{rc}, δ_{ac} and δ_{ec} are respectively the expected deflections of the rudder, aileron and elevator; t_{es} is the instant at which the critical engine is made inoperative; t_{pd} is the time interval between the instant at which the critical engine is failed and the instant at which the pilot recognizes and reacts to the engine failure; t_1 is the instant at which the pilot starts to control the aircraft in order to ensure the aircraft run along the centerline of the runway when the path of the aircraft is returning to the centerline; δ_{r-full} is the limited angle of the rudder deflecting to the side without engine failure; δ_{a1} is the aileron deflection angle when the rudder is full deflected; $N_n \le N_{n\min}$ denotes that the supportive force of the ground effecting on the nose wheel is minor or zero, which indicate that the nose wheel is about to leave the ground or has already left the ground; δ_{e1} and δ_{e2} are the deflections of the elevator expected by the pilot, which make the nose wheel contact the ground slightly [12]; δ_{p-c} and δ_{p-uc} are respectively the throttle skewness of the critical and uncritical engine; δ_{p-io} is the throttle skewness at which the engine is inoperative, δ_{p-io} can be the throttle skewness at which the engine is idle or the throttle fuel is cut off, note that, in order to simulate the added drag force caused by the inoperative engine, δ_{p-io} may be negative (only the equivalent to the added drag force caused by the inoperative engine, in fact the throttle skewness is not negative); δ_{p-matp} is the throttle skewness at which the engine is under the state of maximum available takeoff power or thrust.

The rolling of the aircraft can be eliminated when δ_{a1} is in a certain range, then the requirement of maintain the wing level in 25.149(e) can be met. In the airworthiness compliance flight test of V_{MCG}, the pilot can maintain the wing level by push the control stick to the side without engine failure, and no precise manipulation of aileron control and other special skills is required. Therefore, the specific value of δ_{a1} is not given in this paper. The nose wheel can be maintained to touch the ground slightly when δ_{e1} and δ_{e2} are in a certain range, therefore, this paper will not give the specific value of δ_{e1} and δ_{e2}.

Since the focus in the flight test of V_{MCG} is that whether the maximum run path deviation of the aircraft from the centerline of the runway ($d_{\max}$) is more than 9 meters or not, the manipulation of the pilot after the instant t_1 is not studied in this paper.

5. Dynamics of control actuators

During the takeoff run of the aircraft, the dynamic pressure of the air is low and the hinge moment of control surface is minor. Therefore, we can infer that the deflections of rudder, aileron and elevator are only influenced by the dynamic characteristics of the control surfaces loop. Take the rudder for example, the dynamics of the rudder loop can be expressed as Fig. 3, the dynamic characteristics of aileron and elevator are similar to the rudder [13].

Where, T_r is the time constant of the rudder loop; δ_r is the deflection angle of rudder; s is the Laplace operator.

The thrust of the i th engine is influenced by the throttle skewness, atmospheric properties, aircraft velocity, etc. In order to simulate the change of the thrust when the engine is made inoperative or the throttle skewness changes, an one-order inertia model is used to describe the relationship between T_i and the throttle skewness of the i th engine δ_{p-i} [14, 15], see equation *(25)*. δ_{p-i} is between 0 and 1, where 1 means that the engine is at maximum thrust, and 0 means that the thrust is 0.

$$T_i = \frac{T_{i0}}{T_p s + 1}\delta_{p-i} \tag{25}$$

Where, T_p is the time constant of the engine; T_{i0} is the base thrust of the engine for a certain status.

We can obtain the deflection of the rudder, aileron, elevator and the thrusts of each engines in the airworthiness compliance flight test of V_{MCG} by equation *(21)* to *(25)*. Then, with the aircraft dynamic model introduced in Chapter 3, the “aircraft-pilot” closed-loop motion model of the takeoff run can be established as Fig. 4. With this “aircraft-pilot” closed-loop motion model, the takeoff run in the V_{MCG} airworthiness compliance show can be mathematic simulated precisely.

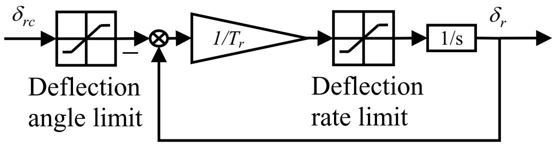

Fig. 3. Dynamic of rudder loop

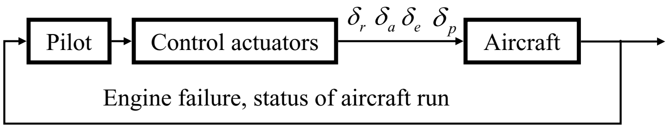

Fig. 4. "Aircraft-pilot" model

6. Steps of V_{MCG} simulation calculation

① According to CCAR-25 25.149(e), set the conditions of V_{MCG} airworthiness compliance flight test, such as the aircraft weight, aircraft CG and the airport height, etc. Set the maximum allowable error δ_{VMCG} in the simulation calculation of V_{MCG}. In this paper, $\delta_{VMCG} = 0.1\text{m/s}$.

② Based on the data of the aircraft aerodynamic, configuration, center of gravity, and engine thrust, etc. Set the nose wheel fixed (the nose wheel rolls along with the aircraft body axis) [12]. The "aircraft-pilot" closed-loop motion model of the V_{MCG} airworthiness compliance show test can be established through the method introduced forward in this paper.

③ Select velocity V_{ES} arbitrarily.

④ Make the critical engine inoperative at the velocity of V_{ES}. And simulate the takeoff run of the aircraft after the critical engine failure based on the "aircraft-pilot" model.

⑤ Based on the mathematic simulation results of step ④, if $d_{\max} > 9\text{m}$, set $V_{ES0} = V_{ES}$ and then increase V_{ES}, repeat step ④ until $d_{\max} < 9\text{m}$, then set $\Delta V_{ES} = V_{ES} - V_{ES0}$ and turn to step ⑥. If $d_{\max} < 9\text{m}$, set $V_{ES0} = V_{ES}$ and then decrease V_{ES}, repeat step ④ until $d_{\max} > 9\text{m}$, then set $\Delta V_{ES} = V_{ES0} - V_{ES}$ and turn to step ⑥. If $d_{\max} = 9\text{m}$, V_{ES} is just the ground minimum control speed under the V_{MCG} airworthiness compliance flight test conditions, that is $V_{MCG} = V_{ES}$, the mathematical simulation ends.

⑥ If $\Delta V_{ES} \le \delta_{VMCG}$, V_{ES} is just the ground minimum control speed under the V_{MCG} airworthiness compliance flight test conditions, that is $V_{MCG} = V_{ES}$, the mathematical simulation ends. If $\Delta V_{ES} > \delta_{VMCG}$ and $d_{\max} < 9\text{m}$, set $V_{ES0} = V_{ES}$ and $V_{ES} = V_{ES} - \Delta V_{ES}/2$, then repeat step ④ and ⑤. If $\Delta V_{ES} > \delta_{VMCG}$ and $d_{\max} > 9\text{m}$, set $V_{ES0} = V_{ES}$ and $V_{ES} = V_{ES} + \Delta V_{ES}/2$, then repeat step ④ and ⑤.

The steps presented above can be shown as Fig. 5.

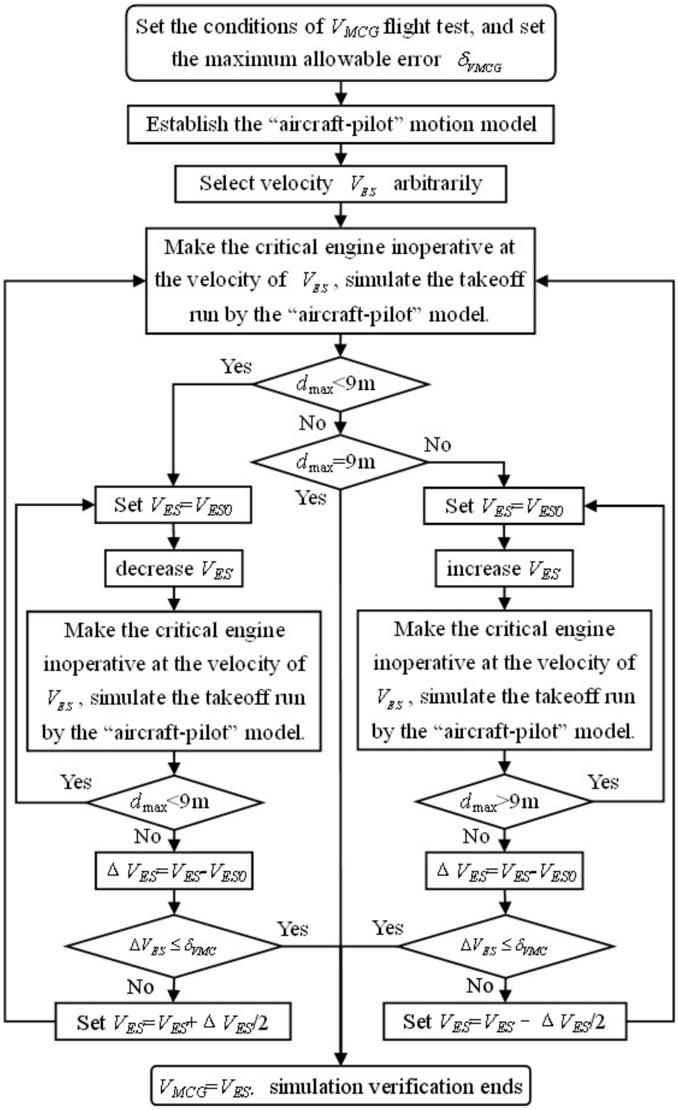

Fig. 5. VMCG simulation verification steps

7. Analysis of simulation experiment

Select a certain type of four-engine aircraft as an example aircraft. There are tow engines installed on both the left and the right wing of the example aircraft. The data needed in the simulation experiment are all derived from the relevant tests or the reliable calculations. By the analysis of the test data, we can know that the outer engine on the right wing of the example aircraft is the critical engine.

Select the sea level as the height of the airport, based on the relevant data of the example aircraft, the "aircraft-pilot" closed-loop motion model of V_{MCG} airworthiness compliance show flight test can be establish. Then, the takeoff run of the example aircraft can be mathematic simulated by the "aircraft-pilot" model.

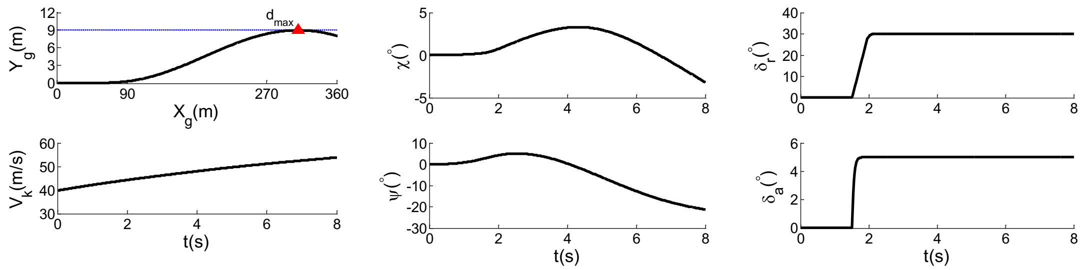

Fig. 6. Results of mathematic simulation experiment of takeoff run

The critical engine is made inoperative at the velocity of 39.8m/s ($t_{es} = 0\text{s}$). 1.5s after the critical engine is failed, the pilot recognizes the critical engine failure and reacts to it immediately ($t_{pd} = 1.5\text{s}$).The results of the mathematic simulation experiment are shown in Fig. 6. During the run of the aircraft after the critical engine is failed, $d_{\max}$ is 9m, therefore, the minimum control speed on the ground of the example aircraft is 39.8m/s under the takeoff conditions, that is V_{MCG}=39.8m/s.

Note that, $d_{\max}$ is seriously influenced by t_{pd} . The shorter t_{pd} is, the earlier the pilot reacts to the critical engine failure, the shorter $d_{\max}$ is, and then the lower V_{MCG} is. t_{pd} is related with the pilot and the situation of the engine failure. The airworthiness standard does not give the specific value of t_{pd} . However, we can consider t_{pd} is 1~3s. The second row of Table 1 shows the V_{MCG} of the example aircraft in sea level calculated by the method presented in this paper when t_{pd} is 1s, 1.5s and 2s respectively. We can find that the longer t_{pd} is, the higher V_{MCG} is.

When t_{pd} is longer enough, the aircraft will not recover to a direction parallel to the centerline before the aircraft leave the ground. Under this condition, the takeoff will be dangerous, and V_{MCG} can not be shown. When $t_{pd} = 2.5\text{s}$, this will happen to the example aircraft.

In order to validate the accuracy of V_{MCG} calculated by the method presented in this paper, the method based on the balance equations of lateral and yaw forces (moments) was used to calculate V_{MCG} of the example aircraft under the same conditions. This method take the speed at which the lateral and yaw forces (moments) are in equilibrium when the rudder full deflected after the critical engine is failed as V_{MCG} [4]. V_{MCG} of the example aircraft calculated by the method of balance equations is 42.2m/s, and the angle of path sideslip is $\beta_k = 4.3^\circ$ when the lateral and yaw forces (moments) are in equilibrium, as shown in the third row in Table 1.

Table 1 VMCG of the example aircraft

t_{pd} (s)	1	1.5	2
V_{MCG} calculated by the method presented in this paper (m/s)	34.0	39.8	50.1
V_{MCG} calculated by the balance equations (m/s)	42.2($\beta_k = 4.3^\circ$)		

As it can be seen from Table 1, there is a certain difference between the V_{MCG} calculated by the method presented in this paper and the method of balance equations. This is because the method presented in this paper has considered the factors such as the time interval between the instant at which the critical engine is failed and the instant at which the pilot recognizes and reacts to the engine failure, the process of the engine thrust decrease, the dynamic of the control surfaces, the aircraft path after the critical engine is failed, etc. All of these factors are important to the airworthiness compliance show of V_{MCG}. However, the method of the balance equations does not take these factors into account. Therefore, V_{MCG} calculated by the method presented in this paper could be accurate.

8. Conclusions

The dynamic motion model of the aircraft in the takeoff run is introduced in this paper. And the model of the pilot's operation for the airworthiness compliance show of V_{MCG} is presented. Then, the "aircraft-pilot" closed-loop motion model is established. Based on this "aircraft-pilot" model, a mathematic simulation method to calculate V_{MCG} is presented in this paper. This method has considered all forces and moments act on the aircraft during the takeoff run after the critical engine is failed. And this method has also considered the factors such as the time interval between the instant at which the critical engine is failed and the instant at which the pilot recognizes and reacts to the engine failure, the thrust decrease of the inoperative engine, the dynamics of the control actuators, etc. Therefore, V_{MCG} calculated by this method could be accurate.

V_{MCG} calculated by the method presented in this paper can be used to guide the modification and improvement of the aircraft airworthiness design. And it can also be used to guide the airworthiness compliance flight test of V_{MCG}.

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